Math2121 (Spring 2012-2013) **Tutorial Note 4**

Matrix Operations;
Inverse, Transpose and Elementary Matrices; Determinants

Key Questions

- How to perform matrix multiplication?
- How to find inverse by elementary row operations?
- What is the transpose of a matrix?
- What are elementary matrices?
- What is the determinant of a 2×2 matrix?
- How to compute determinants by means of cofactor expansion?
- What are the useful computational rules of determinants that avoid tedious expansion?
- Why do we need to study determinants?
- When $n = 2$, how is $A(T(E))$ related to T and E? Also when $n = 3$, how is $V(T(E))$ related to T and E? Here T is linear, A denotes area and V denotes volume.

Problem 1. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

Compute Bu, vA, BA and vBu.

Solution.

Problem 2. Let

$$A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix},$$

compute A^{-1} by elementary row operations.

Solution.

Problem 3. Find A^T and B^T, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Solution.

Problem 4. Let $C = [c_{ij}]_{n\times n} \in M_{n\times n}(\mathbb{R})$, we define the trace of C to be

$$\operatorname{Tr} C = c_{11} + c_{22} + \cdots + c_{nn},$$

i.e., $\operatorname{Tr} C$ is the sum of all diagonal entries. Let $A \in M_{n\times k}(\mathbb{R})$ and $B \in M_{k\times n}(\mathbb{R})$, then AB and BA are square matrices, show that

$$\operatorname{Tr}(AB) = \operatorname{Tr}(BA).$$

Solution.

Problem 5. Show that

$$\begin{vmatrix} 1 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & 1 & 6 \end{vmatrix} = 12 \quad \text{and} \quad \begin{vmatrix} 3 & 2 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 6 & 0 & -1 \\ -6 & 3 & 1 & 0 \end{vmatrix} = -49.$$

Solution.

Problem 6. Show that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

Solution.

Problem 7. Show that

$$\begin{vmatrix} a^2 & bc & c^2+ca \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Solution.