Math2121 (Spring 2012-2013) **Tutorial Note 1**

Linear Systems; Echelon and Reduced Echelon Forms;
Basic and Free Variables

Key Questions

- What are echelon and reduced echelon form?
- How to solve a system of linear equations?
- What are free and basic variables of a linear system?

Problem 1. Find the general solution of the systems:

$$\begin{cases} x+y+2z+w=5 \\ x+y+2z+6w=10 \\ x+2y+5z+2w=7 \end{cases}$$

Also identify the free variables and basic variables.

Solution.

Problem 2. Determine if the following system has a nontrivial solution. Then describe the solution set.

$$\begin{cases} 3x_1+5x_2-4x_3=0 \\ -3x_1-2x_2+4x_3=0 \\ 6x_1+x_2-8x_3=0 \end{cases}$$

Solution.

Problem 3. Let A be a matrix with the following reduced row echelon form

$$U = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Also, let $a_1, a_2, a_3, a_4 \in \mathbb{R}^4$ be the columns of A, if $a_1 = \begin{bmatrix} -3 \\ 5 \\ 2 \\ 1 \end{bmatrix}$ and $a_2 = \begin{bmatrix} 4 \\ -3 \\ 7 \\ -1 \end{bmatrix}$, find a_3 and a_4.

Solution.